Applied Nonparametric Econometrics

Theoretical Problem Set #1 (Density Estimation)

1. Is it possible to derive a Silverman type rule-of-thumb bandwidth if $f(x)$ was is Cauchy?

2. Provide an intuitive explanation of why the difficulty factor, $\int f^{(2)}(x)^2\,dx$, is a reasonable measure of difficulty. Is this measure scale invariant? What types of densities are 'easy' to estimate? Why (Terrell, 1990 is a good reference for more on this issue)?

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Theoretical Problem Set #2 (Inference about the Density)

1. For the test of symmetry (Ahmad and Li, 1997b), show that the conditions for $H_n(x_i, x_j) = k\left(\frac{x_i - x_j}{h}\right) - k\left(\frac{x_i + x_j}{h}\right)$ being a degenerate U-statistic hold.

2. For the test of correct parametric specification (Fan, 1994), show that for a Gaussian kernel and an assumed normal distribution that

$$\begin{aligned}\widehat{ISE}_n &= \frac{1}{n(n-1)h}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} k\left(\frac{x_i - x_j}{h}\right) + \frac{1}{2\sqrt{\pi}(h^2+\widehat{\sigma}^2)^{1/2}} \\ &\quad - \frac{2}{n\sqrt{2\pi}(h^2+\widehat{\sigma}^2)^{1/2}}\sum_{i=1}^{n}\exp\left[-\frac{(x_i-\widehat{\mu})^2}{2\left(h^2+\widehat{\sigma}^2\right)}\right]\end{aligned}$$

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Theoretical Problem Set #3 (Regression Estimation)

1. If the joint density of two variables (Y and X) is normal, show that the conditional expectation ($E(Y|X = x)$) must be linear in x.

2. Show that when $m(x) = \alpha + \beta x$ is linear in x, then the local-linear estimator is unbiased (note that you should not use $h = \infty$ in this problem).

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Theoretical Problem Set #4 (Testing in Regression)

1. For the test of correct parametric specification (Li and Wang, 1998), show that the conditions for $H_n\left(x_i, x_j\right) = \widehat{u}_i \widehat{u}_j k\left(\frac{x_i - x_j}{h}\right)$ being a degenerate U-statistic hold.

1. For the test of correct parametric specification (Ullah, 1985), show that the following hold for the wild bootstrapped residuals:

   (a) $E\left[u_i^*\right] = 0$

   (b) $E\left[\left(u_i^*\right)^2\right] = \widehat{u}_i^2$

   (c) $E\left[\left(u_i^*\right)^3\right] = \widehat{u}_i^3$